Dear Editor,

In this revision, we have addressed the concerns of the reviewers properly. In summary, our major changes to the original version lie in the following aspects:

(1) We have given a strict NP-hard proof for the problem;

(2) We have devised a polynomial-time algorithm for the skeleton partition problem when the genus of the mesh model, the degree of each node and the number of portioned components in the skeleton graph are bounded by a small constant integer.

(3) A super computer with a large number of cores is called to verify the performance of our proposed method. Supplement materials including the experiment data and codes for each task are also submitted for reviewers’ reference. Instructions on how to use the code is also provided.

(4) We have added some more discussion on the use of the 1D Laplacian Skeleton.

We believe that this paper is helpful for readers of geometric modeling and computational geometry with applications in 3D printing.

(5) For a comparison job, we have reprinted a set of models; each model is oriented in its best gesture for saving printing materials, where the orientation is determined by Meshmixer, a 3D printing software provided by Autodesk (leader of the CAD/CAM software market).

The following are the details replies to the reviewers.

Associate Editor

Comments to the Author:

While the goal of support-free printing is well appreciated by all reviewers, the reviews, including my own reading, identified a few critical issues that, in my opinion, make the paper unsuited for being considered further for TVCG. The key issues are:

1. Limited scope. The method works only for shapes that can be represented by curve skeletons. But there is a more fundamental limitation, based on my own reading. The paper seeks to optimize the angle span of the curve skeleton, with the hope that the surface triangles will have a similar angle span. This is a wrong assumption in general. The direction of the curve skeleton can be quite different from the orientation of the mesh triangles, particularly when the thickness of the shape varies significantly (e.g., consider the skeleton of a fat cone). So the method really only works for objects made up of cylinders with slowly varying diameter. This significantly limits the application of the work in practice.

We admit that our work does not consider the case of a fat cone in which the direction of surface triangles might not parallel to their corresponding skeleton piece. However, cylindrical shapes whose triangles are almost parallel to their skeleton pieces are ubiquitous in both organic forms (human body and trees, etc) and man-made artifacts. See the model decomposition using cylinders in Ref. [42].

[42] Y. Zhou, K. Yin, H. Huang, H. Zhang, M. Gong, and D. Cohen-Or, “Generalized cylinder decomposition”, ACM Trans. Graph., vol. 34, no. 6, pp. 171:1–171:14, Oct. 2015.

2. Poor algorithmic choices. The method consists of ad-hoc heuristics that lack rigor and are poorly justified. I agree with Reviewer 1 that there is no direct connection between the stated problem and the bin-packing problem, which puts the NP-hardness claim in doubt.

We have added a strict proof for the NP-hardness of the problem.

I want to add that the problem of finding maximum angle span of a set of vectors, which is a key technical component of this method (Section 4, "Skeleton partition"), is in fact equivalent to the well-studied problem of finding the smallest enclosing circle to a set of points on a plane (after conformal mapping), whose optimal solution can be found efficiently (in linear time).

This is the case for static point set, and in expected sense; in the worst-case sense, this problem reduces to sorting, which takes O(*n* log *n*) time. But, in essence, this problem is computing the diameter of a planar set of points with dynamic insertions (since a vector can be mapped to a point on a projection plane, and the vectors join in one by one instead of all at once), whether a worst-case logarithmic query time exists or not is unknown at present; see the following reference from Prof. Orourke’s open problem list.

<http://cs.smith.edu/~orourke/TOPP/P12.html>

But, in an expected sense, a randomized process that add the vectors one by one (upon a random permutation of the vectors) can result in an expected updated time of O(1). Refer to Chapter 4 of the following reference:

de Berg M, van Kreveld M, Overmars M, Schwarzkopf O. Computational geometry: algorithms and applications. 3rd ed. Berlin, Germany: Springer-Verlag; 2008.

3. Lack of sufficient evaluation, particularly w.r.t. the choices of various parameters.

We proposed a polynomial time algorithm for special cases of the skeleton partition problem. We evaluate the result with the help of a super computer with 100 cores.

Reviewer Comments

Please note that some reviewers may have included additional comments in a separate file. If a review contains the note “see the attached file” under Section III A – Public Comments, you will need to log on to ScholarOne Manuscripts to view the file. After logging in, select the Author Center, click on the “Manuscripts with Decisions” queue and then clicking on the “view decision letter” link for this manuscript. You must scroll down to the very bottom of the letter to see the file(s), if any. This will open the file that the reviewer(s) or the Associate Editor included for you along with their review.

Reviewer: 1

Recommendation: Author Should Prepare A Major Revision For A Second Review

Comments:

Support material waste is a big problem for popular low-cost 3D printers, and decomposing the shapes into parts without any support can help save both material and time. While most of the previous works focused on solid shapes, this paper took 3D shell models as input, maybe because printing shell models can save more materials? (This was not clearly explained in the paper)

Support material saving for solid models has been done by Ref. [6]. Therefore, we focus on shell models instead. Further, there is a great difference between these two types in choosing the support-free printing direction: for a solid pyramid, a large facet is usually set as the base; while for a hollow one, it is possible that no printing direction guarantees support-free fabrication. We have added some more explanation about this in the revised version.

Inspired by the 1D representation of organic models, i.e., skeleton, the 3D shape decomposition problem was converted into a graph partition problem. This conversion is new and creative. The overall results shown in the paper look good and Table 1 does show that both material and time are highly saved after the partition.

My main concern is the way of how the graph partition problem is solved. First of all, in line 49-52 in page 3, it is claimed in the paper that the decomposition problem is exactly the bin packing problem and thus NP-hard. However, this equivalence is not so clear to me. First, the constraint for each sub-fork is that the angle between any two arcs is small than 2\theta, which is quite different from packing items with different weights into bins of specific capacity. Second, sub-forks are not arbitrary set of arcs but connected subgraphs, which makes the problem further different. I think there are several more differences between those two problems, to the authors should give a convincing proof of the equivalence instead of just claiming the problem is NP-hard. As a result, it also makes question whether such complicated and greedy method is needed to such the original decomposition problem.

We have added a strict proof, linking the problem of skeleton partition to the clique cover problem (instead of bin-packing), which is NP-hard.

The main strategy for solving the decomposition is to determine the number of mesh components first and then the total cutting length. For the first part, a very greedy method is introduced even with some probability-based schemes for choosing the growing arc and terminating the growing. For the second part, with all those cases, the constraints can still be violated and there is no guarantee for getting results with good quality to some extent.

We agree that it is very hard to obtain an optimal solution by our approach, yet after a comparison with the result of a super computer, we find that our result is only slightly worse than the optimum.

We remark that with the help of the training and learning scheme running for a large number of iterations, we can refine the quality of result by assigning the arcs to different subgraphs.

For the evaluation section, I think more experiments are need to evaluate different steps of the method and see how those heuristics affect the final results. More shapes need to be tested especially for Fig. 12, which is mainly used to show how close the results found by the method are to the optimal solutions. The testing examples shown here are too simple and not representative enough. Moreover, since the method is mainly designed for shell models and can also be applied to solid models, I’m wondering how the thickness would affect the decomposition results and how the results look like for solid models.

Here are some more detailed comments:

1. Line 45-48 in page 3 (left): "our objective is ...", this sentence is incorrect comparing to constraint 1&2.

Revised.

2. Line 58 in page 3 (right): b(H\_i) should be area(b(H\_i)). How can use set the threshold value?

The value can be determined experimentally. For example, in FDM setting, depending on the printing speed and layer thickness, a simple experiment can be carried on to determine the value. Normally, a base of 22 mm is sufficiently large.

3. Line 25 in page 5 (left): if I understand correctly, e\_k is not on B\_{i,j}

Revised.

4. Line 39 in page 5 (left): the way of getting the probability seems need a lot of efforts by running the procedure so many times

Yes, it needs a large number of iterations, but it does not take that much running time.

5. When cutting the parts, how to make sure each part would be stable during the printing?

In a printing process, the bottom of a part is firmly attached to the printing platform (in many 3D printers, the platform contains many small holes such that the bottom of the part is firmly hold by on the platform). In addition, the material (e.g., plastic) itself has certain cohesive force and elastic force, which guarantee that the part can be printed in a stable manner without falling down.

Additional Questions:

1. Which category describes this manuscript?: Research

2. How relevant is this manuscript to the readers of this periodical? Please explain your rating under Public Comments below.: Very Relevant

1. Please explain how this manuscript advances this field of research and/or contributes something new to the literature. What do you see as the strongest and the weakest aspect of the paper? : This paper proposed a new algorithm for partitioning a 3D model into least number of parts for 3D printing without any support. Comparing to previous works, this proposed method is the first one aimed for shell models. The idea of converting the problem to partitioning a 1D skeleton graph into subgraphs is novel and interesting.

However, the proposed method is too complicated and consists of a bunch of heuristics, which makes it hard for me evaluate how effective the method is and which part contributes most. Moreover, it's claimed in the paper several times that the optimization problem is NP-hard but without any proof.

2. Is the manuscript technically sound? Please explain your answer under Public Comments below.: Yes

1. Are the title, abstract, and keywords appropriate? Please explain under Public Comments below.: Yes

2. Does the manuscript contain sufficient and appropriate references? Please explain and list missing references under Public Comments below.: References are sufficient and appropriate

3. Does the introduction state the objectives of the manuscript in terms that encourage the reader to read on? Please explain your answer under Public Comments below.: Yes

4. How would you rate the organization of the manuscript? Is it focused? Is the length appropriate for the topic? Please explain under Public Comments below.: Satisfactory

5. Please rate the readability of the manuscript. Explain your rating under Public Comments below.: Readable - but requires some effort to understand

6. Should the supplemental material be included? (Click on the Supplementary Files icon to view files): Does not apply, no supplementary files included

7. If yes to 6, should it be accepted:

Please rate the manuscript. Explain your choice.: Fair

Reviewer: 2

Recommendation: Author Should Prepare A Minor Revision

Comments:

Overall, the proposed algorithm appears to be sound and the results are convincing, however, I do have several questions/concerns:

While the assumption of input can be decomposed into cylindrical parts is valid for human/animal characters and some mechanical parts, 3D printed shapes are often much more diverse and complex. In addition, high frequency details (e.g. spikes on cactus) may not be captured by skeletons and could still cause printing problems.

We agree that high frequency tiny details that may not be captured by skeletons can cause problems. However, the printing material itself has some inherent cohesive and elastic forces, which can pull an overhang region to some extent if the tiny detail is a short one and the overhanging angle is not strictly horizontal. This effect is more obvious as the printing layer thickness is small enough (e.g., 0.1 mm). Refer to the last model in Figure 12, albeit the existence of some defects on model surface, we find that the spikes on cactus can be printed in a support-free manner without hurting its (boundary) surface quality. Further, we have also conducted a new experiment using a shape with short spikes in Figure 3. We have added some comments on this in the revision.

Sometimes, minimizing supporting material may lead to longer printing time. It is much faster to print the 8-shape in Fig 12 with the shape laying flat on the printing platform with some support structures than having it standing up without support as indicated in the figure. Some discussions on this would be helpful.

Yes, the printing time not only depends on the amount of material, but also the path planning strategy. In this paper we focus on the problem of minimizing the material used.

In the revision, we have put each model (including the 8-shape model) in its most material saving gesture with the aid of Meshmixer software. The printing experiment for the original models has been conducted for all the new gestures. The 8-shape model lies in that way due to its hollow interior.

The description of mesh partition in section 4 is a bit hard to follow. It would be helpful if more illustrations are provided. Part of the algorithm also seems very ad-hoc. For example, the last paragraph of section 4 describes a quick fix for unprintable mesh partition induced by a printable skeleton partition.

Won't this cause undesirable tiny mesh segments? A more in-depth look of this technique and its effects would be very helpful. Is it the cause that the number of mesh partitions are more than the number of skeleton partitions (table 1)? Is it the cause of the over segmentation of both the rare end and the head of the deer mesh in Figure 1?

Yes. Under the requirement of support-free fabrication, our approach may incur the problem of over segmentation. Allowing a very moderate amount of support material, the tinny segmentation can be avoided. We have added more description on this part.

It would be great if the number of skeleton arcs is also reported in table 2.

Revised.

Additional Questions:

1. Which category describes this manuscript?: Research

2. How relevant is this manuscript to the readers of this periodical? Please explain your rating under Public Comments below.: Relevant

1. Please explain how this manuscript advances this field of research and/or contributes something new to the literature. What do you see as the strongest and the weakest aspect of the paper? : The paper proposes an mesh partitioning technique such that each part can be 3D printed without support structures. Support structures cost both money and time in fabrication, and it is important to study algorithms that could avoid or minimize the use of support structures.

The algorithm is based on the assumption that the input 3D model can be decomposed into cylindrical parts, and such decomposition can be inferred from the input shape's skeleton. The proposed Monte Carlo approach uses a greedy approach to group connected skeleton arcs satisfying printability constraints together. The skeleton partition induces a partition of the actual mesh. The algorithm is repeated with randomly chosen seed arcs and the best partition with minimum partitions and cutting perimeter is returned.

The paper includes a number of fabricated results demonstrating the algorithm can significantly reduce the amount of support needed to 3D print a given shape.

2. Is the manuscript technically sound? Please explain your answer under Public Comments below.: Appears to be - but didn't check completely

1. Are the title, abstract, and keywords appropriate? Please explain under Public Comments below.: Yes

2. Does the manuscript contain sufficient and appropriate references? Please explain and list missing references under Public Comments below.: References are sufficient and appropriate

3. Does the introduction state the objectives of the manuscript in terms that encourage the reader to read on? Please explain your answer under Public Comments below.: Yes

4. How would you rate the organization of the manuscript? Is it focused? Is the length appropriate for the topic? Please explain under Public Comments below.: Satisfactory

5. Please rate the readability of the manuscript. Explain your rating under Public Comments below.: Readable - but requires some effort to understand

6. Should the supplemental material be included? (Click on the Supplementary Files icon to view files): Does not apply, no supplementary files included

7. If yes to 6, should it be accepted: As is

Please rate the manuscript. Explain your choice.: Fair

Reviewer: 3

Recommendation: Reject

Comments:

The paper claims that the proposed work focuses on shell model however I do not see how the algorithm handles shell models specifically different from volumetric models. The revised paper should explain better the differences between decomposing a shell model and that of a volumetric model, and the revised paper should also provide better explanation on which part of the algorithm is designed to handle shell model. In addition, the work closest to this paper is the work done by Hu et al. 2014 “Approximate pyramidal shape decomposition", which should be compared to in the revised version of this paper.

We noticed this paper, and it is one of our references. But the problem is that we have totally different focuses, Hu’s paper emphasizes on “pyramidal shape decomposition”, while out paper allows a shape resembling a tree or a torus, etc. Secondly, printing a solid pyramid in a support-free manner can be done as the large base is set at the bottom, but this is not the case for a shell model. With these differences, it is not reasonable to make a comparison. We have added some discussion on this in the revision (Figure 2).

The paper only compares the proposed segmentations to the models without segmentation. However, even these comparisons do not seem to be fair. For example, the knots model requires a lot of supporting materials because the model is not at its optimal orientation. If the model is lay flat, then the results will be very different. It is also unclear why the printing direction should be a user parameter. Couldn't the proposed method determine the optimal printing direction?

In the revision, we have put the models in its most material-saving gesture with the aid of Meshmixer software (AutodeskTM).

Some parameters used in the proposed algorithm is also not explained. For example, how detailed or how abstract is the 1-D skeleton?

The Laplacian skeletons we used are provided by the authors of the following paper:

O. K. Au, C. Tai, H. Chu, D. Cohen-Or, and T. Lee, “Skeleton extraction

by mesh contraction,” ACM Trans. Graph., vol. 27, no. 3, 2008.

The skeleton contains many small pieces; each corresponds to a fairly small strip of mesh vertices. We have added some discussion on this when introducing the Laplacian Skeleton.

The main drawback of the paper is perhaps the choice of methods used in designing the proposed segmentation algorithm. For example, it is unclear why using 1-D skeleton would be a better choice than the traditional clustering based methods. These 1-D skeletons tend to lose a lot of geometric information thus do not provide sufficient details to guarantee a support-free partition (as shown in Fig 8). The paper claims that this rarely happens but it is also hard to justify what "rare" really means. For a model contains significant surface noise or surface texture, this does happen often. In addition, many objects, such as a cut, do not have a natural 1D skeleton.

In the revision, we have added some discussions and experiments on the use of 1D Laplacian Skeleton and the impact of surface noises (small spikes).

Several examples of poor choice of methods can also be found in determining the cutting planning for mesh partition. In page 5, lines 24~42 explain how the concavity can be used to determine the cutting plane by introducing the idea of insignificant concavity and uses the vertices R(v) with significant concavity and also R(u) from the neighboring vertices with significant concavity to determine the cutting plane. It is unclear why this particular definition of concavity is used and why R(u) from the neighboring vertices should be included. Furthermore, the paper requires that the cutting plane should also avoid cutting through the other skeleton, but I do not think that this is always possible.

Segmentation according to organic features is still not mature in a semantic sense. Using concavity to guide a cut is an intuitive way of cutting a mesh, and including the neighboring vertices into R(u) may increase the chance of a nice cut that satisfies the constraints while meeting the objective.

Yes, we require that the cutting plane should also avoid cutting through the other skeleton, and this is not always feasible. And if so, a compromise is allowed. Instead of maintaining all the constraints, our algorithm respects all constraints as much as possible; and when a relaxation is needed, the relaxation is done as little as possible.

Even though the proposed method tries to minimize the cut length, the resulting 3D prints still have visible seams, in particular the model shown in Fig. 1 near the back&tail of the dear. This might be the result of restricting the cuts on/near skeleton vertices or the fact that minimizing the number of component has higher priority than minimizing the cut length in the proposed framework. In any case, I would like to see quality improvement in the revised version.

Although the length of the cuts accounts for the aesthetic of the assembled model, we find that the number of cuts affects more: it begs for a precise matching and gluing for many times if the number of cuts is large. Given the condition that the current desktop 3D printers are stable and precise enough, no deformation occurs during the printing processed, and matching two parts along a short or a long curve means little difference to the surface quality. More discussion on this has been added to the revision.

For a quality improvement of the paper, we first show that the problem of skeleton partition is NP-hard by a strict proof; and then we show that the problem of optimal skeleton partition can be solved in polynomial time under some special conditions. We have implemented our proposed algorithm for this special case and summarized the results in the revision.

Finally, it is not clear if Monte Carlo method is the best framework to use as well. The paper presents a purely random approach without any domain knowledge to guide the search.

The search is guided by a weighting scheme that puts more probability on a choice that can reduce the number of cuts. In solving such an intricate problem involving various parameters, Monte Carlo Method with a large number of iterations is a good choice. To guarantee that the method can converge to a nice result within limited number of iterations, we apply a training-and-learning procedure, which helps accelerate the searching process.

Additional Questions:

1. Which category describes this manuscript?: Application

2. How relevant is this manuscript to the readers of this periodical? Please explain your rating under Public Comments below.: Relevant

1. Please explain how this manuscript advances this field of research and/or contributes something new to the literature. What do you see as the strongest and the weakest aspect of the paper? : The paper explores the problem of partitioning a water-tight model into support-free components for FDM 3D printers. The main idea is to partition the 1-D skeleton tree to ensure that all tree branch can be printed without support (for a given printing direction). The segmentation of the skeleton is then transferred to partition the mesh while several heuristics are used to provide better aesthetic features and minimize the cut length.

This paper is one of the few works in the literature that explores the idea of generating support-free decomposition. The paper is clearly written and easy to follow even though there are quite a few language problems and some technical details are not very well explained. The weakest part of the paper is probably the technical contribution. Many technical choices are not well motivated and better options probably exist.

2. Is the manuscript technically sound? Please explain your answer under Public Comments below.: No

1. Are the title, abstract, and keywords appropriate? Please explain under Public Comments below.: Yes

2. Does the manuscript contain sufficient and appropriate references? Please explain and list missing references under Public Comments below.: References are sufficient and appropriate

3. Does the introduction state the objectives of the manuscript in terms that encourage the reader to read on? Please explain your answer under Public Comments below.: Yes

4. How would you rate the organization of the manuscript? Is it focused? Is the length appropriate for the topic? Please explain under Public Comments below.: Satisfactory

5. Please rate the readability of the manuscript. Explain your rating under Public Comments below.: Readable - but requires some effort to understand

6. Should the supplemental material be included? (Click on the Supplementary Files icon to view files): Does not apply, no supplementary files included

7. If yes to 6, should it be accepted:

Please rate the manuscript. Explain your choice.: Poor